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Scheduling Maintenance Operations Which Cause Age - Dependant Failure Rate Changes

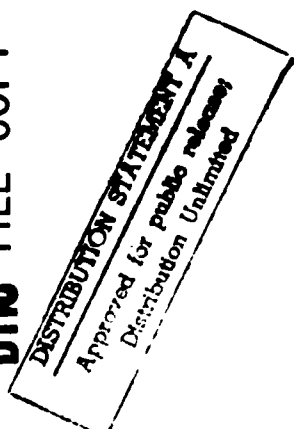
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SCHEDULING MAINTENANCE OPERATIONS WHICH
CAUSE AGE-DEPENDENT FAILURE RATE CHANGES*

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ABSTRACT

→ We consider optimization of schedules for maintenance or repairs, in order to minimize long term average operating cost or to maximize availability. The novelty here is that the failure rate after a maintenance operation is a function of the system's previously expended lifetime. This generalizes earlier work by others on the simpler case where the future rate depends only on the number of previous repairs, but not on the times when they took place. The underlying lifetime distributions are assumed to have the Weibull form and two classes of maintenance strategies are considered.

The first case optimizes a set of successive maintenance intervals T_1, T_2, \dots, T_N , and the number N , where a replacement by a new system is made at $t_N = \sum_{i=1}^N T_i$. For the specified model, we show that the optimal times T_i exists and are ordered as $T_1 \geq T_2 > \dots \geq T_N$. Additional properties of the optimal solutions are proved, and others conjectured.

In the second case, the period of periodic maintenance is optimized numerically. The main contribution in that case is the formulation of the new failure rate model, and the efficient organization of the optimization calculations.

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1. INTRODUCTION

This work extends the maintenance models and policy optimizations described in the references. In particular, the ideas in Nguyen and Murthy (to appear) and Goldstein (1980) were strong influences.

We consider optimization of the schedule of times at which preventive maintenance will be carried out on a system subject to stochastic failures. Several different criteria are used to judge the quality of a schedule, including long-term average operating costs per unit time, and various definitions of availability. The effect of maintenance is represented as a reduction in the subsequent failure rate. One of our contributions is an approach for incorporating the influence of past operating time on the new failure rate functions which applies after a maintenance operation.

A component or system with a failure time \underline{t} (random variable) can be described by its failure rate or hazard rate $r(t)$:

$$r(t)dt = P[t < \underline{t} \leq t + dt \mid \underline{t} > t] \quad (1)$$

where the cumulative distribution function is related to $r(t)$ as

$$F(t) = 1 - \exp\left[-\int_0^t r(z) dz\right] \quad (2)$$

Figure 1 shows two forms used later for the reduction in failure rate at maintenance times t_1 . In case a), maintenance reduces $r(t_1)$ to zero, but subsequent growth rate is greater than before the maintenance. That growth rate will be a function of the previous operating time t_1 . In case b), the growth rate of $r(t)$ is constant, but maintenance reduces r at t_1 by an amount dependent on t_1 .



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We define $r_i(t)$ between maintenance times as:

$$r(t) = r_i(t - t_{i-1}) \quad t_i < t \leq t_i \quad (3)$$

We consider $r_i(t)$ which are increasing in t , with numerical results based on the Weibull forms

$$r_i(t) = \lambda_1 t^\alpha \quad (4)$$

or

$$r_i(t) = \lambda_1(t + t_{i-1})^\alpha - \Delta_i$$

(equivalent to $r(t) = \lambda_1 t^\alpha - \Delta_i$)

These $r_i(t)$ are also increasing functions of the maintenance intervals $T_j = (t_j - t_{j-1})$ for $j < i$.

In some models we permit minor repairs when the system fails. After such a repair, the failure rate $r(t)$ evolves as if no failure had occurred. This model is often used for complex equipment in which the next failure is unlikely to be related to the component which was just repaired. From this point of view, the number of failures and minor repairs between maintenance times, $N(t_{i-1}, t_i)$, has

$$E[N(t_{i-1}, t_i)] = \int_{t_{i-1}}^{t_i} r_i(t) dt \quad (5)$$

For both cases in Fig. 1, the system wears out ($r(t) \rightarrow \infty$ as $t \rightarrow \infty$) so complete replacement (renewal) is eventually required.

Section 2 considers a problem where minor repairs are permitted between maintenance times, and costs are assigned to renewal, minor repair and maintenance operations. With replacement at t_N , we seek the best N and t_1, t_2, \dots, t_{N-1} to minimize the average cost per unit time when renewal cycles are repeated indefinitely. This is mathe-

matically equivalent to maximizing the steady-state availability, by suitable reinterpretation of the costs.

Section 3 assumes that each failure requires a complete renewal, and that the maintenance times are equally spaced (often the case due to work schedules). Cycle availability A_c is defined as the fraction of time that the system is up (not in maintenance or replacement operations) during one renewal cycle. The maintenance period is sought to maximize

$$P[A_c > \delta] , \quad 1 < \delta < 0 \quad (6)$$

2. SCHEDULES WITH MINOR REPAIRS

We assume that renewal occurs at t_N with a cost C_R ; maintenance occurs at T_1, T_2, \dots, T_{N-1} at a cost of C_M each time; and minor repairs occur at each failure with a cost C_r . The expected cost per renewal cycle is then

$$L(N, \vec{T}) = C_R + (N-1) C_M + C_r \sum_{j=1}^N \int_0^{T_j} r_j(t) dt \quad (7)$$

where $\vec{T} = (T_1, T_2, \dots, T_N)$.

For these purposes, all three corrective operations are assumed to be instantaneous, corresponding to practical durations which are small compared to system uptime. The renewal cycle duration is then

$$D(N, \vec{T}) = t_N = \sum_{j=1}^N T_j \quad (8)$$

and the long-term average cost per renewal cycle is

$$C(N, \vec{T}) = L(N, \vec{T})/D(N, \vec{T}) \quad (9)$$

Theorem 1

If i) $r_i(t)$ is strictly increasing in t, T_1, \dots, T_{i-1}

$$\text{ii) } \frac{\partial^2 r_i(t)}{\partial T_m \partial T_n} \geq 0 \quad \text{all } i, t, m, n$$

Then the necessary conditions

$$L_i(N, \bar{T}) \triangleq \frac{\partial L(N, \bar{T})}{\partial T_i} = C^*(N, \bar{T}), \quad i = 1, 2, \dots, N \quad (10)$$

have a solution corresponding to a global minimum of $C(N, \bar{T})$ with respect to the T_i . Saddle points, but not a relative maximum, are also possible solutions to (10).

Proof: Evaluation of the Hessian matrix at the stationary point shows that a relative maximum cannot exist for any \bar{T} . $C(N, \bar{T})$ is a continuous function of the T_i and it approaches infinity as $\|\bar{T}\| \rightarrow 0$ or $\rightarrow \infty$.

Theorem 2

If the $r_i(t)$ have the Weibull form

$$r_i(t) = \lambda_0 [1 + \varepsilon \sum_{j=1}^{i-1} T_j] t^\alpha \quad (11)$$

then the optimal T_i are ordered as

$$T_1^* \geq T_2^* \geq \dots \geq T_N^* \quad (12)$$

Proof: This result is derived by showing that if $T_i = a < T_{i+1} = b$, the average cost will be reduced by changing to $T_i = b$, $T_{i+1} = a$, while keeping all other T_j unchanged. The comparison is facilitated by the fact that this interchange does not affect t_N or $r_k(t)$ for $k < i$ or $k > i+1$.

Another optimal-solution property which aids in computational problems is summarized in

Theorem 3:

If a) $r_i(t)$ are strictly increasing in t , and

$$b) \frac{\partial r_i}{\partial T_m} = \frac{\partial r_j}{\partial T_n} \quad \text{all } n < i, \quad m < j$$

Then the optimal T_i satisfy

$$r_i(T_i^*) < r_j(T_j^*) ; \quad i < j . \quad (13)$$

The Weibull $r_i(t)$ in Theorem 2 satisfies the conditions of Theorem 3. The result is exemplified in Fig. 1(a) where the peaks get higher going to the right.

Proof: This follows from examination of the following relation, based on (10).

$$L_i(N, \bar{T}) - L_{i+1}(N, \bar{T}) = 0 .$$

The preceding theorems all deal with the case of fixed N . We also want to find the optimum number of maintenance operations ($N-1$). Clearly, interesting cases will require $C_R > C_M$ and $C_R > C_T$. The following properties have been observed in all numerical examples we have studied, but sufficient conditions and proofs have not yet been found.

Conjecture 1. $t_N^* = \sum_{i=1}^N T_i^*$ is an increasing function of N .

Conjecture 2. The minimal cost as a function of N is convex, as in Fig. 2, so that an optimal N^* can be found by getting

$$\bar{T}_{(1)}^*, \bar{T}_{(2)}^*, \dots, \text{ until} \\ C(N+1, \bar{T}_{(N+1)}^*) > C(N, \bar{T}_{(N)}^*).$$

The model considered here is closely related to one in Nguyen and Murthy. They have $r_i(t)$ as functions of i only (not on t_i). In that simpler case, equation (13) is replaced by $r_i(T_i^*) = r_j(T_j^*)$ and Conjecture 2 has been proved.

We now turn to a slightly different problem, using (9) for the criterion, but with case (b) of Fig. 1 to represent the maintenance effect. In particular

$$r_i(t) = \lambda(t + t_{i-1}) - \gamma t_{i-1} ; \quad 0 < \gamma < \lambda \quad (14)$$

Fig. 1(b) has $\gamma = \lambda/2$. Expressing (14) in terms of the intervals T_j we have

$$r_i(t) = \lambda t + (\lambda - \gamma) \sum_{j=1}^{i-1} T_j \quad (15)$$

so that $r_i(t)$ is an increasing function of t and the T_j ($j < i$).

Here, a necessary condition for the best T_i 's based on partial derivatives of (9), is

$$\gamma(T_i - T_j) = 0 ; \quad i, j = 1, 2, \dots, N. \quad (16)$$

Thus, for this model

$$T_1^* = T_2^* = \dots T_N^* = T^* \quad (17)$$

Substitution of (17) into $C(N, \bar{T})$ leads to

$$T^* = [-(\lambda - \gamma)(N-1) + \sqrt{(\lambda - \gamma)^2(N-1)^2 + 8\lambda(C_R - (N-1)C_M)/C_r}]/2\lambda \quad (18)$$

In this example we have also found an optimal N in every numerical example. Although the joint optimization of N and \bar{T} reduces here to a two parameter optimization (N and T^*), existence and uniqueness of the overall minimum have not yet been proved.

All of the preceding results can be interpreted with respect to an availability criterion. If C_R , C_M and C_r are viewed as the mean times required, respectively, for replacement, maintenance and minor repair, then the long-term availability is

$$A[N, \bar{T}] = \frac{D[N, \bar{T}]}{L[N, \bar{T}] + D[N, \bar{T}]} = [1 + C(N, \bar{T})]^{-1}$$

Thus, A is maximized here by minimizing C using the techniques described earlier.

3. PERIODIC MAINTENANCE -- NO MINOR REPAIRS

In this model every failure is followed by a replacement (renewal). The replacement time has an exponential distribution with mean value $1/\mu$, and maintenance operations have fixed duration d . The operating time between maintenances is u , and failure may occur during either system operation or maintenance. Failure rate reduction, of the form of case (b) in Fig. 1, occurs at the end of each maintenance. The use of equal operating intervals between maintenances is motivated by work-rule convenience and the results in the previous section for a related problem where that structure is optimal.

The maintenance effect is introduced differently here, but for the example of linearly increasing hazard rate, there is a direct

connection with the models of Section 2. Here we say that maintenance reduces $r(t)$ by a factor:

$$r(t_i^+) = g r(t_i^-) ; \quad 0 < g < 1 \quad (19)$$

Thus, for $t_{i-1} < t < t_i$,

$$\begin{aligned} r(t) &= r_1(t) - \Delta_i \\ \Delta_{i+1} &= (1-g) [r_1(t_i) - \Delta_i] ; \quad \Delta_1 = 0 \end{aligned} \quad (20)$$

For the case of $r_1(t) = \lambda t$, the maintenance effect becomes

$$\Delta_{i+1} = (1-g) [i \lambda s - \Delta_i] \quad (21)$$

where $s = (u + d)$. Using the notation of (4), this is equivalent to

$$\begin{aligned} r_i(t) &= \lambda t + \theta_{i-1} ; \quad i = 2, 3, \dots \\ \theta_i &= \lambda s \sum_{j=1}^i g^j \end{aligned} \quad (22)$$

where the θ_i are increasing functions of u (period between maintenances).

We want to find u to maximize the probability that cycle availability is above an acceptable level. With T_u and T_d representing the total up and down times in a renewal cycle, we have

$$A_c = \frac{T_u}{T_u + T_d}$$

and the quantity to be maximized is given in (6). T_d includes all of the time spent in maintenance before the failure, plus the replacement time T_R after the failure.

It is convenient to define E_n^u and E_n^d as the events that failure occurs, respectively, during the n th operating interval or the n th maintenance interval. Then

$$P[A_c \geq \delta] = \sum_{n=1}^{\infty} \left[\frac{U_n P_n(\delta) + D_n Q_n(\delta)}{U_n + D_n} \right] \quad (23)$$

$$\text{where } U_n = P[E_n^u], \quad D_n = P[E_n^d] \quad (24)$$

$$P_n(\delta) = P[A_c \geq \delta | E_n^u], \quad Q_n(\delta) = P[A_c \geq \delta | E_n^d]$$

Goldstein (1980) found analytical expressions for (23) in the simpler case where the durations of each operating period, maintenance period and replacement were independent random variables with exponential distributions. Only numerical evaluation seems possible for the present model.

The piecewise definition of $r(t)$ in (20) can be converted to a corresponding definition for the lifetime probability density which can be used for numerical integration to evaluate the quantities in (23) for each n . (The summation is truncated when terms become small.) For $t_{n-1} < t < t_n$ we have

$$f(t) = r(t) R(t)$$

$$R(t) = R_n(t) \prod_{j=1}^{n-1} R_j(t_j)$$

$$R_j(t) = \exp\left[-\int_{t_{j-1}}^t r(\tau) d\tau\right] : t_{j-1} < t \leq t_j$$

Also, we note that using T_R for the replacement time after failure

$$P_n(\delta) = \int_{t_{n-1}}^{t_{n-1}+u} f(t) P\{T_R < [t(1-\delta)/\delta - (n-1)d/\delta]\} dt$$

and a similar expression applies for $Q_n(\delta)$.

Numerical examples have shown that this approach is practical to get $P[A_c > \delta]$ for fixed u , and that a maximum can be found by varying the value of u . Very small u will introduce excessive time spent on maintenance; large u will delay maintenance until it is too late to be effective. Figure 3 shows examples for $P[A_c > 0.9]$ where the optimum maintenance period is 0.07 of the mean lifetime, or 0.18 of the mean lifetime, for different parameter choices.

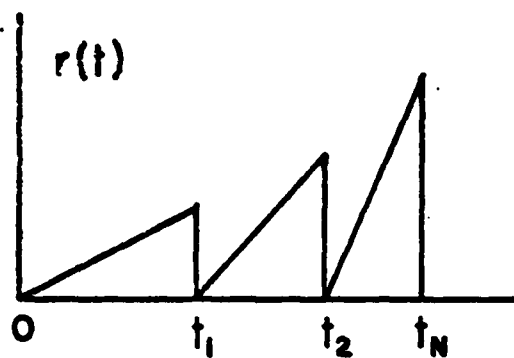
4. CONCLUSIONS

Optimization of maintenance schedules has been considered for problems where maintenance reduces the subsequent failure rate by an amount depending on the previous operating time of the system. The analysis is more difficult than in similar problems where the effect of maintenance depends only on the number of previous maintenances. Some useful properties of optimal solutions have been demonstrated, and others are yet to be verified.

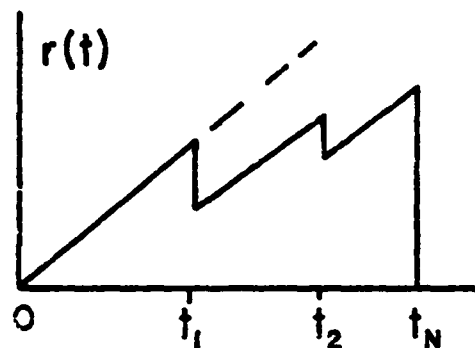
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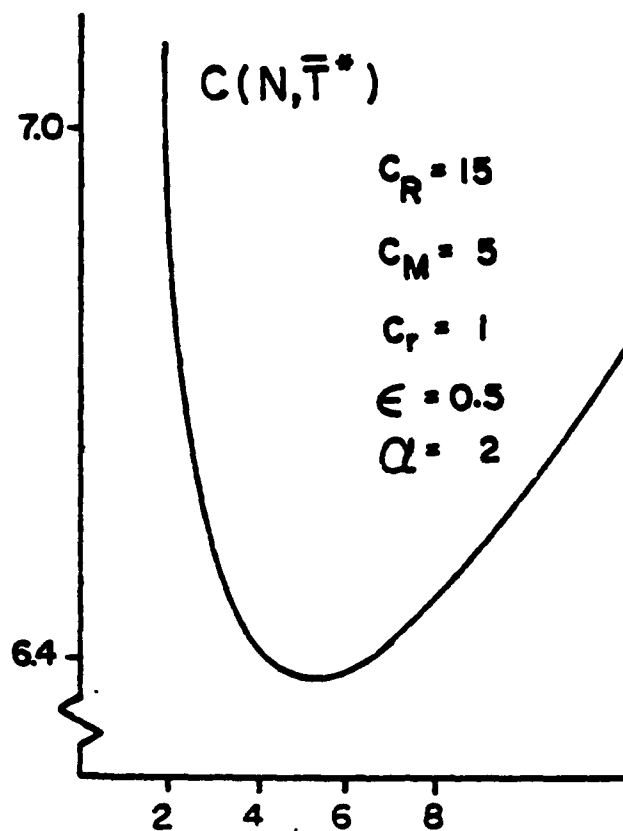
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Case a



Case b



Numerical Example, Convex $C(N, \bar{T})$

Figure 2

Figure 1
Maintenance-Dependent Failure Rates

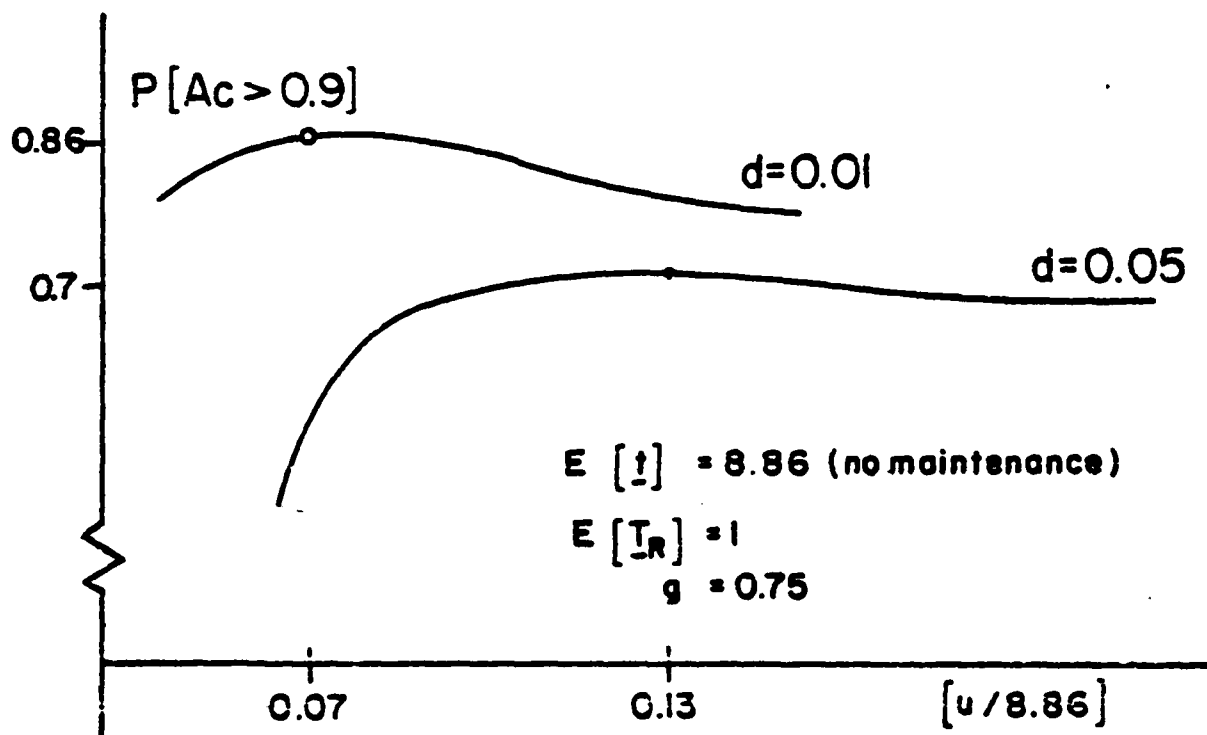


Figure 3 - Optimization of Periodic
Maintenance Interval u

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